DOES $\sum_{n=1}^{\infty} \frac{n+1}{3n^2-1}$ CONV/DIV ?

LARGER NUMER

$$\frac{n+1}{3n^2-1} > \frac{n}{3n^2} = \frac{1}{3n}$$

SMALLER DENOM

$$\cancel{\frac{n+1}{3n^2-1}} \qquad \cancel{\frac{1}{3n}}$$

$$0 < \frac{1}{3n} < \frac{n+1}{3n^2-1}$$

LARGER NUMER

$$\overbrace{\frac{n+1}{3n^2-1} > \frac{n}{3n^2-1}} > \frac{n}{3n^2}$$

SMALLER DENOM

$\llcorner$ NOTE: $3n^2-1=0$ ONLY AT $n=\pm\frac{1}{\sqrt{3}}$

not $\geq n=1$

$\boxed{\text{TO BE CONTINUED}}$

$$\sum_{n=1}^{\infty}\frac{1}{3n} = \frac{1}{3}\sum_{n=1}^{\infty}\frac{1}{n} \quad \text{DIV} \quad (p\text{-SERIES TEST})$$

$$p=1\leq 1$$

SO $\sum_{n=1}^{\infty}\frac{n+1}{3n^2-1}$ DIV (COMPARISON TEST)

(COMP TEST)

DOES $\sum_{n=1}^{\infty} \frac{2^{2n}}{3^n - 1}$ CONV/DIV?

P-SERIES
GEOMETRIC
DIVERGENCE
COMPARISON
INTEGRAL

$$\lim_{n\to\infty} \frac{2^{2n}}{3^n - 1} = \lim_{n\to\infty} \frac{2^{2n}(\ln 2)2}{3^n \ln 3}$$

$\frac{\infty}{\infty}$ $\frac{\infty}{\infty}$

$$= \lim_{n\to\infty} \frac{4^n (2\ln 2)}{3^n \ln 3}$$

$$= \lim_{n\to\infty} \left(\frac{4}{3}\right)^n \left(\frac{2\ln 2}{\ln 3}\right) = \infty$$

SO $\sum_{n=1}^{\infty} \frac{2^{2n}}{3^n - 1}$ DIV (DIV TEST)

OR $$\lim_{n\to\infty} \frac{2^{2n}}{3^n - 1} \cdot \frac{\frac{1}{3^n}}{\frac{1}{3^n}} = \lim_{n\to\infty} \frac{\frac{2^{2n}}{3^n}}{1 - \frac{1}{3^n}}$$

$$= \lim_{n\to\infty} \frac{\left(\frac{4}{3}\right)^n}{1 - \frac{1}{3^n}}$$ $\frac{\infty}{1-0} \to \frac{\infty}{1} \to \infty$

$$= \infty$$

DOES $\sum_{n=1}^{\infty} \frac{1}{n+2^n}$ CONV/DIV?

$$0 < \frac{1}{n+2^n} < \frac{1}{2^n}$$

$\sum_{n=1}^{\infty} \frac{1}{2^n}$ CONV (GEOMETRIC SERIES TEST)
(GEO TEST)

$|r| = \frac{1}{2} < 1$

SO, $\sum_{n=1}^{\infty} \frac{1}{n+2^n}$ CONV (COMP TEST)

$\cancel{0 < \frac{1}{n+2^n} \leq \frac{1}{n}}$ $\cancel{0 < \frac{1}{n} < \frac{1}{n+2^n}}$ FALSE

$\sum_{n=1}^{\infty} \frac{1}{n}$ DIV (p-SERIES TEST)

$p = 1 \leq 1$

BUT NO CONCLUSION ABOUT $\sum_{n=1}^{\infty} \frac{1}{n+2^n}$ IS POSSIBLE

DOES $\sum_{n=1}^{\infty} \frac{n-1}{3n^2+1}$ CONV/DIV ?

$$\frac{n-1}{3n^2+1} < \frac{n}{3n^2+1} < \frac{n}{3n^2}$$

$$\frac{n-1}{3n^2+1} \qquad \frac{n}{3n^2} = \frac{1}{3n}$$

Σ DIV

$$0 < \frac{n-1}{3n^2+1} < \frac{1}{3n}$$

→ NO CONCLUSION IS POSSIBLE FROM COMPARISON TEST

IS THERE A CONSTANT k SUCH THAT

$$\underbrace{\frac{1}{k}\,\frac{1}{3n}}_{\Sigma\text{ DIV}} < \frac{n-1}{3n^2+1} \ ?$$

COMP TEST SAYS Σ DIV

$$\frac{n-1}{3n^2+1} > \frac{\frac{1}{2}n}{3n^2+1} > \frac{\frac{1}{2}n}{4n^2}$$

NOTE: $n-1 > \frac{1}{2}n$ IF $\frac{1}{2}n > 1$, $n > 2$

$3n^2+1 < 4n^2$ IF $1 < n^2$, $n > 1$

CONSIDER

$$\sum_{n=1}^{\infty} \frac{n-1}{3n^2+1} = 0 + \frac{1}{13} + \sum_{n=3}^{\infty} \frac{n-1}{3n^2+1}$$

$$\frac{n-1}{3n^2+1} > \frac{\frac{1}{2}n}{3n^2+1} > \frac{\frac{1}{2}n}{4n^2} = \frac{1}{8n} > 0$$

IF $n \geq 3$

$$\sum_{n=3}^{\infty} \frac{1}{8n} = \frac{1}{8} \sum_{n=3}^{\infty} \frac{1}{n} \text{ DIV } \text{ (P-SERIES TEST)}$$

$$p = 1 \leq 1$$

$$\text{SO } \sum_{n=3}^{\infty} \frac{n-1}{3n^2+1} \text{ DIV } \text{ (COMP TEST)}$$

$$\text{SO } \sum_{n=1}^{\infty} \frac{n-1}{3n^2+1} \text{ DIV}$$

$$= 0 + \frac{1}{13} + \sum_{n=1}^{\infty} \frac{n-1}{3n^2+1}$$